Current reversal with type-I intermittency in deterministic inertia ratchets

Woo-Sik Son,[1,*] Inbo Kim,[1,†] Young-Jai Park,[1,‡] and Chil-Min Kim[2,§]
[1]*Department of Physics, Sogang University, Seoul 121-742, Korea*
[2]*National Creative Research Initiative Center for Controlling Optical Chaos, Department of Physics, Paichai University, Taejon 302-735, Korea*

(Received 25 August 2003; published 11 December 2003)

The intermittency is investigated when the current reversal occurs in a deterministic inertia ratchet system. To determine which type the intermittency belongs to, we obtain the return map of velocities of particle by using stroboscopic recordings, and by numerically calculating the distribution of the average laminar length $\langle l \rangle$. The distribution follows the scaling law of $\langle l \rangle \propto \epsilon^{-1/2}$, the characteristic relation of type-I intermittency.

DOI: 10.1103/PhysRevE.68.067201 PACS number(s): 05.45.Ac, 05.40.Fb, 05.45.Pq, 05.60.Cd

In the last 10 years there has been an increasing interest in the ratchet system [1]. It has been studied theoretically and experimentally in many different fields of science, e.g., molecular motors [2], new methods of separation of particles [3], and current-voltage rectification in asymmetric superconducting quantum interference devices [4]. More recently, there have been several works on classical [5] and quantum [6] domains, Josephson-junction array [7], and etc.

The ratchet system [8] is generally defined as a system that is able to transport particles in a periodic structure with nonzero macroscopic velocity (although on average no macroscopic force is acting) [9]. For directional motion of a particle with no macroscopic force, or unbiased fluctuation in a periodic structure, the system has to be driven away from thermal equilibrium by an additional deterministic [10] or stochastic perturbation [11]. Besides the breaking of thermal equilibrium, the breaking of spatial inversion symmetry is usually required further for directional motion [12]. For this, the ratchet-shaped potential has been introduced. It has also shown that the directional motion can exist in the presence of spatially symmetric potential with external perturbation that is time asymmetric [13].

Many works on the ratchet system have been mainly limited to the overdamped case [14]. In these works, systems of interest are related to the microscopic scale in which thermal fluctuations or noises play a dominant role. Recently, Jung *et al.* have considered the effect of finite inertia [15]. By considering the inertia term, the dynamics can exhibit both regular and chaotic behaviors. They have shown that deterministic chaos, to some extent, mimics the role of noise, and there exist multiple current reversals as the amplitude of external driving is varied. Thereafter, this system has been called "deterministic inertia ratchets," or "deterministic underdamped ratchets" [16,17].

In a recent paper, Mateos showed that the origin of the current reversal in the deterministic inertia ratchets may be related to a bifurcation from the chaotic to the regular regime. He also mentioned the diffusion property in the intermittent chaotic regime [18]. After his work, it has been conjectured that the mechanism of current reversal may be related to a crisis bifurcation in which a chaotic state suddenly becomes periodic [16]. Later, it was also shown that the current reversal, in the same system, can occur even in the absence of bifurcation from chaotic to regular regime on the other parameter ranges [17]. However, up to our knowledge, any further work on the detailed characteristic of the intermittent behavior has not been reported.

On the other hand, the Pameau and Manneville types of intermittency are mainly classified into types I, II, and III by the structure of the local Poincaré map, $v_{n+1}=v_n+av_n^2+\epsilon$, $v_{n+1}=(1+\epsilon)v_n+av_n^3$, and $v_{n+1}=-(1+\epsilon)v_n-av_n^3$, respectively [20]. These types of intermittency are characterized by characteristic relations, $\langle l \rangle \propto \epsilon^{-1/2}$ for type-I, and $\langle l \rangle \propto \epsilon^{-1}$ for type-II and type-III, where $\langle l \rangle$ is the average laminar length. Here, the parameter ϵ in type-I intermittency is the channel width between the diagonal line and the local Poincaré map, while $1+\epsilon$ in type-II and type-III is the slope of the local Poincaré map around the tangent point.

The aim of this Brief Report is to investigate the characteristic of the intermittent behavior in the deterministic inertia ratchets. We take the same system as in Ref. [18], and explicitly show that the type-I intermittency exists when a current reversal occurs from the chaotic to the regular regime.

Now, let us consider a system in which a particle moves in ratchet potential, subjected to time-periodic driving and damping. The equation of motion is written as

$$\frac{d^2x}{dt^2}+b\frac{dx}{dt}+\frac{dV}{dx}=a\cos(\omega t), \tag{1}$$

where b is the friction coefficient and ω and a are the frequency and the amplitude of the external driving, respectively. Here, $V(x)$ in Fig. 1 is the ratchet potential, and is given by

$$V(x)=C-\frac{\sin 2\pi(x-x_0)+0.25\sin 4\pi(x-x_0)}{4\pi^2\delta}, \tag{2}$$

*Electronic address: dawnmail@physics3.sogang.ac.kr
†Electronic address: ibkim@physics3.sogang.ac.kr
‡Electronic address: yjpark@sogang.ac.kr
§Electronic address: chmkim@mail.paichai.ac.kr

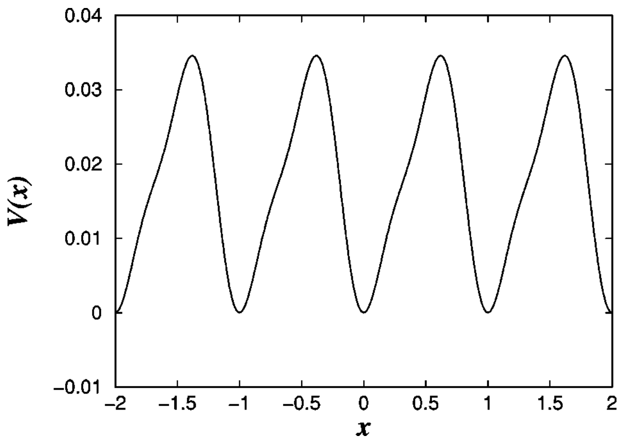

FIG. 1. The asymmetric periodic potential $V(x)$ with parameters $C=-(\sin 2\pi x_0+0.25\sin 4\pi x_0)/4\pi^2\delta$, $x_0=-0.19$, and $\delta \simeq 1.614\,324$ as in Ref. [18].

where C and x_0 are introduced to show that the potential has a minimum at $x=0$ with $V(0)=0$, and $\delta=\sin(2\pi|x_0|)+\sin(4\pi|x_0|)$.

Among the three dimensionless parameters in Eq. (1), we vary the parameter a, and fix $b=0.1$ and $\omega=0.67$ throughout this Brief Report as in Ref. [18]. The equation of motion, Eq. (1) is nonlinear. The inertia term allows the possibility of chaotic orbits. We solve this system numerically, using fourth-order Runge-Kutta algorithms. Because of sensitivity in the chaotic system above, we calculate all values and parameters including δ with double precision.

In this system, there exists a bifurcation from the chaotic to the regular regime when the current reversal takes place through varying the parameter a [18]. As shown in Fig. 2, the particle shows intermittent chaotic behavior in the bifurcation region. This particle moves almost regularly in a negative direction, and occasionally shows chaotic burst. Furthermore, to study the behavior of the system during the current reversal, we plot a bifurcation diagram on the velocity of a particle. To obtain this bifurcation diagram, we take stroboscopic recordings of the first derivative of x at times $t=k\tau$ where k is the positive integer, and τ is the period of the external driving, $\tau=2\pi/\omega$. In Fig. 3 of the bifurcation diagram, velocities of the particle are plotted on the parameter range of $a\in(0.074\,000\,000,0.086\,000\,000)$. Note that when we plot this diagram, we freely take the initial point as $(x_0,v_0)=(0,0)$ at time $t=0$ for each parameter a because there only exists a global attractor in the range above, and the long initial transient data is dropped before plotting. The result is analogous to Fig. 2(a) in Ref. [18]. On the other hand, there also exists another current reversal phenomenon in the absence of bifurcation from the chaotic to the regular regime in $a\in(0.140\,000\,000,0.170\,000\,000)$. In this range, there are coexisting attractors and hysteresis [17,19].

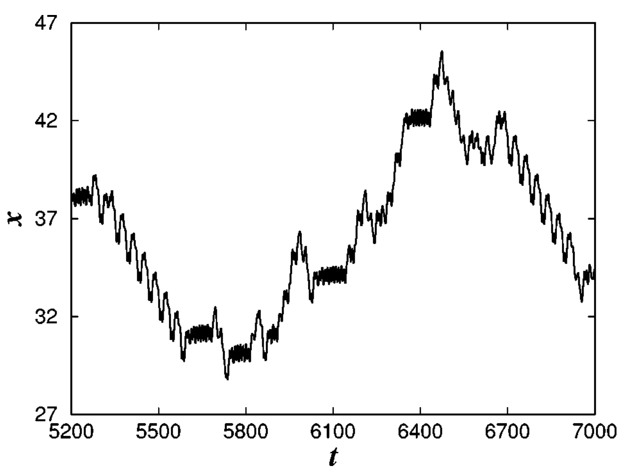

FIG. 2. For $b=0.1$ and $\omega=0.67$, we show the intermittent chaotic trajectory of the particle at $a=0.080\,910\,000$ during current reversal.

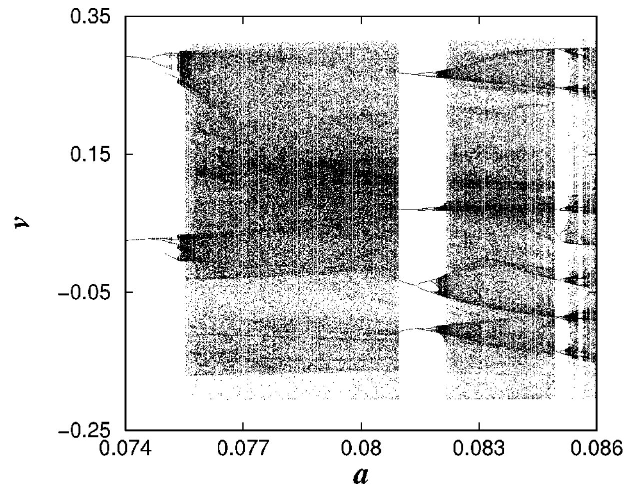

FIG. 3. The bifurcation diagram with varying the parameter a, other parameters are fixed, $b=0.1$ and $\omega=0.67$.

In Fig. 3, two fixed points of velocities plotted in the bifurcation diagram at $a=0.074\,000\,000$ correspond to the *regular positive current of the particle having period two*, while four fixed points of velocities at $a=0.080\,990\,000$ correspond to the *regular negative current of the particle having period four* [18]. During this current reversal, there is a period-doubling route to chaos, as shown in Fig. 3. The bifurcation from the chaotic to the regular regime takes place at critical value a_c, just above $a=0.080\,947\,429$ [21]. After this bifurcation, a periodic window, corresponding to the regular negative current, emerges.

To determine which type the intermittency above belongs to, we numerically obtain the return map, $f^4(v_n)$, which shows the relation between the velocities of particle, v_n and v_{n+1}, after 4τ time interval elapses, with external driving period τ, $\tau=2\pi/\omega$. We use the value of τ with double precision because of the sensitivity of the chaotic system. In Fig. 4, we plot the return map at just below a_c. Figure 5 is an enlargement of Fig. 4 in the vicinity of $v_n=-0.031$. The points on the diagonal line in the return map correspond to the states of $v_{n+1}=v_n$, i.e., four periodic motion.

As shown in Fig. 5, the return map is nearly tangent to the diagonal line at the parameter a just below a_c. When a particle remains in a narrow channel, the trajectory shows an almost regular periodic behavior. After the particle escapes from this narrow channel, the trajectory shows a chaotic

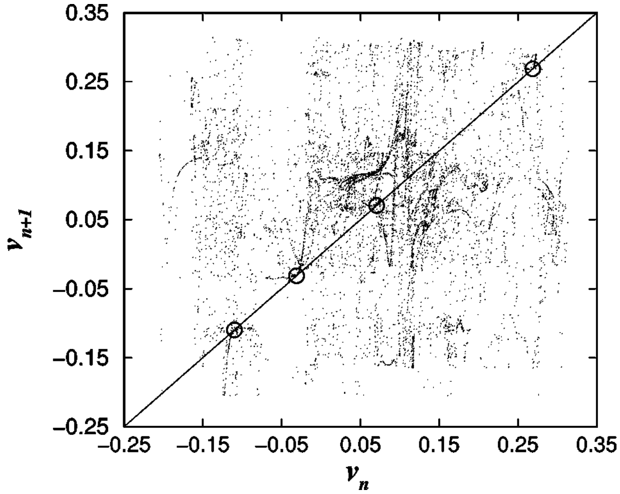

FIG. 4. The return map is plotted at the parameter $a = 0.080\,947\,429$ just below a_c. In the vicinity of -0.110, -0.031, 0.071, and 0.270, the return map is nearly tangent to the diagonal line. Four open circles indicate the nearly tangent regions.

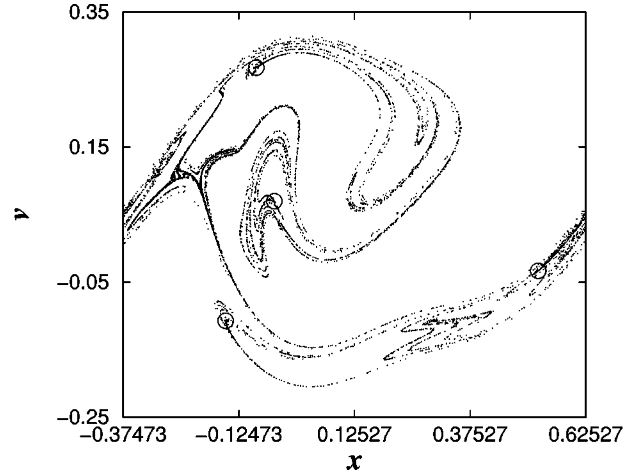

FIG. 6. The phase portrait of two attractors of the ratchet equation for $b=0.1$ and $\omega=0.67$: one is the chaotic attractor for $a = 0.080\,947\,429$, just below a_c and the other is the period four attractor for $a=0.080\,990\,000$, represented by the center of four open circles.

burst, and then goes back to the channel. This process keeps repeating. As a increases from the value of just below a_c to a_c, the channel width narrows more and more so that the particle spends most of the time in there. After all, the return map crosses the diagonal line, when a becomes larger than a_c, and two crossing points, corresponding to the stable and unstable fixed points, are made. Among these points, the stable fixed point corresponds to the regular negative current of the particle. Like this, the intermittency emerges, before the return map undergoes a tangent bifurcation. This analysis agrees well with the result shown in Fig. 6, which is similar to that of Mateos [18].

In Fig. 6, one of two attractors is the chaotic one for $a = 0.080\,947\,429$, just below a_c, and the other is the period four attractor for $a=0.080\,990\,000$, corresponding to the regular negative current. Note that these two attractors are obtained by confining the dynamics in one potential well that includes $x=0$, i.e., in the range of $x \in (-0.374\,734\,43, 0.625\,265\,57)$. The latter consists of four points at the phase space. In the chaotic attractor, a particle spends most of the time in the vicinity of four point attractors corresponding to the regular negative current. Once in a while it intermittently moves in a chaotic way.

So far, we have investigated the type of intermittency qualitatively. For a quantitative characterization of the type of intermittency, we survey the scaling law for duration of the laminar state. That means the particle remains in the narrow channel, as shown in Fig. 5. We calculate the average laminar length $\langle l \rangle$ by averaging the iterated numbers of the return map at the laminar state, as a changes from the range of below a_c to a_c. Numerically, we take the laminar state as

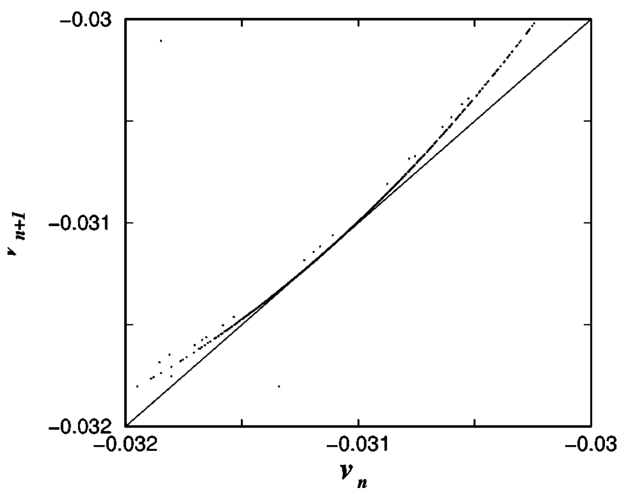

FIG. 5. The enlargement of Fig. 4 in the vicinity of $v = -0.031$. The return map seems nearly tangent to the diagonal line, but it is not exactly tangent. The channel width between the return map and the diagonal line is in the order of 0.000 001.

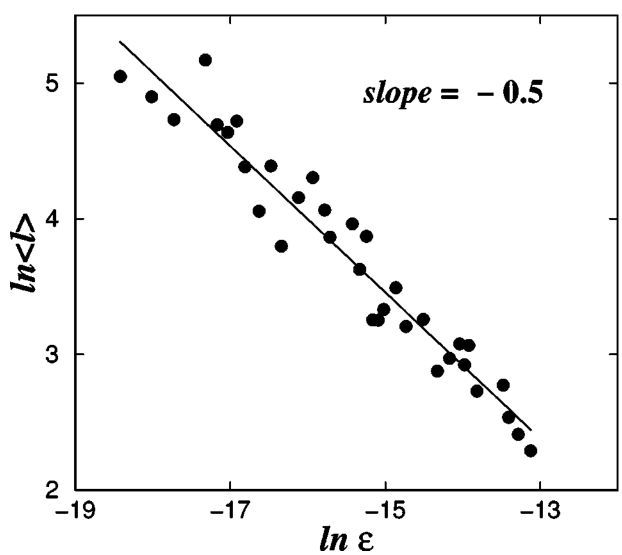

FIG. 7. The distribution of the average laminar length with varying the parameter a. It follows the scaling law of $\langle l \rangle \propto \epsilon^{-1/2}$ with $\epsilon = a - a_c$.

the situation that the difference between v_{n+1} and v_n is less than 0.0003. The result has been shown in Fig. 7. The distribution of the average laminar length follows the scaling law of $\langle l \rangle \propto \epsilon^{-1/2}$ with $\epsilon = a - a_c$. It agrees with that the distribution of the average laminar length of the type-I intermittency is in the form $\langle l \rangle \propto \epsilon^{-1/2}$ if there are no external noises [20,22,23]. Therefore, the intermittency that exists before the bifurcation taking place from the chaotic to the regular regime is the type-I intermittency.

In conclusion, we have investigated the mechanism of the current reversal in deterministic inertia ratchets. By numerically obtaining the return map of velocities, and by using the scaling law giving the characteristic relation of intermittency, we have explicitly shown that the type-I intermittency exists when the current reversal occurs from the chaotic to the regular regime.

The authors thank D.-U. Hwang, J.-W. Ryu, and M.-U. Kim for valuable discussions. This work was supported by the Creative Research Initiatives of the Korean Ministry of Science and Technology. Two of us (W.-S.S and Y.-J.P.) acknowledge support from the Sogang University Research Grants in 2003.

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